StudyForge
DIGITAL CURRICULUM

US Geometry

Type: Online

Course Description:

Geometry offers a rich and engaging subject to explore with students! Students will ease into the course by initially focusing on plane Euclidean geometry, which allows for both synthetic and analytical approaches, and enables students to further develop their logical reasoning skills. Then, exploring congruence, similarity, and symmetry through geometric transformations and exploring and applying theorems about triangles and quadrilaterals will help students develop their problem-solving abilities. While trigonometry opens doors to real-world applications and deeper mathematical understanding, circle geometry provides a robust theoretical environment to hone problem-solving and reasoning skills even further. Finally, by integrating geometry with algebra, students will see the power of mathematical connections and provide them with new problem-solving tools for the future. Ultimately, by presenting geometry as a creative, practical and challenging yet rewarding subject, students are given a nutrient-rich environment in which to learn and master the subject.

Units:

- Review: Warm-up to Geometry
- Lines and Angles
- Triangles
- Polygons and Transformations
- Trigonometry
- Circles
- Measurement
- Reasoning and Proofs
- Statistics
- Probability

By the end of the course, students will be able to:

- Gain a solid understanding of geometric objects and relations
- Develop a keen understanding of the relationships between geometric objects and properties and their interaction with the real world
- Comprehend the importance of logical reasoning, use it to solve problems, and apply it to prove conjectures and theorems

Assessments:

- Unit Note Packages
- Lesson Practice Questions
- Quizzes (mid-unit and end-of-unit)
- Unit Review Assignment

- Unit Practice Test
- Unit Test
- Unit Projects
- Final Exam

Requirements:

- Scientific Calculator for assessments (handheld or online)
    - Note: Every practice question has a built-in Desmos graphing calculator.
- A small student geometry kit (compass, protractor, ruler, etc.) will be helpful but is not required

Learning Standards Overview:

Common Core Standards Covered	Standards Description
Unit 1: Review: Warm-up to Geometry	
HSG-CO.A.1	• Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
HSG-GPE.B.5	• Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
HSG-GPE.B.6	• i.[1] Find the point on a directed line segment between two given points that partitions the segment in a given ratio. • ii. Note: A derivative of this standard is finding the midpoint of two points or a line segment as well as using the midpoint formula.
HSG-CO.D.12	• i. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). • ii. Copying a segment; copying an angle; bisecting a segment; bisecting an angle. • iii. Constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
8.G-B.8	• Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
8.EE-6	• Construct an equation using the slope m and the y-intercept b in the form of $y=mx+b$.
Unit 2: Lines and Angles	

[1] Some standards have been broken into sections (i., ii., iii., etc.) in order to show which part is covered in each lesson in the second table set and to indicate any additional aspect(s) of the lesson.

HSG-CO.C.9	• i. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; • ii. points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
7.G-B.5	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
HSG-CO.D.12	• i. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). • ii. Copying a segment; copying an angle; bisecting a segment; bisecting an angle. • iii. Constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
Unit 3: Triangles	
HSG-CO.C.9	• i. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; • ii. points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
HSG-CO.C.10	• i. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent. • ii. The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. • iii. Note: Triangle Inequality Theorem is also covered. • iv. Note: Angle-Side Inequality Theorem is also covered. • Note: Proofs covered in Unit 8.
HSG-SRT.B.5	• Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
HSG-CO.B.7	• Use the definitions of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
HSG-CO.B.8	• Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow the definition of congruence in terms of rigid motions.
HSG-GPE.B.6	• Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
HSG-C.A.3	• i. Construct the inscribed and circumscribed circles of a triangle. • ii. and prove properties of angles for a quadrilateral inscribed in a circle.

HSG-SRT.B.4	• i. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely. • ii. The Pythagorean Theorem proved using triangle similarity. • iii. Other Theorems covered include: the length of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the lengths of the two segments of the hypotenuse, and; the centroid of the triangle divides each median in the ratio 2:1. • Note: Proofs covered in Unit 8.
HSG-SRT.C.8	• i. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. • ii. Note: Ratios could include an altitude drawn to the hypotenuse of a right triangle.
Unit 4: Polygons and Transformations	
HSG-CO.C.11	• i. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, • ii. opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. • iii. Note: Investigation of interior and exterior angles is also covered.
HSG-GPE.B.7	• Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
HSG-CO.B.6	• Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
HSG-CO.B.7	• Use the definitions of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
HSG-SRT.A.1	• Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
HSG-SRT.A.2	• Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
HSG-SRT.A.3	• Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
HSG-CO.A.2	• i. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs.

	• ii. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). • iii. Note: Lesson includes dilations.
HSG-CO.A.3	• Describe a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
HSG-CO.A.4	• Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
HSG-CO.A.5	• Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Unit 5: Trigonometry	
HSG-SRT.D.9	• Derive the formula *A=1/2absin(C)* for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
HSG-SRT.C.6	• Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
HSG-SRT.B.4	• i. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely. • ii. The Pythagorean Theorem proved using triangle similarity. • Note: Proofs covered in Unit 8.
HSG-SRT.C.7	• Explain and use the relationship between the sine and cosine of complementary angles.
HSG-SRT.C.8	• i. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. • ii. Note: Lesson includes derivation of trigonometric ratios. • iii. Note: Lesson includes using Pythagorean triples to solve problems.
HSG-SRT.D.10	• Prove the Laws of Sines and Cosines and use them to solve problems.
HSG-SRT.D.11	• Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Unit Lesson 6: Circles	
HSG-C.A.2	• i. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles. • ii. The radius of a circle is perpendicular to the tangent where the radius intersects the circle.

HSG-C.A.3	• Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
HSG-CO.D.13	• Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
HSG-C.A.4	• Construct a tangent line from a point outside a given circle to the circle.
HSG-C.B.5	• i. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. • ii. Note: Defining degree measure is included in lesson. • iii. Note: Applying the proportional relationship between the area of a sector and area of a circle to solve problems is included in lesson. • iv. Note: Applying the proportional relationship between the arc length and circumference to solve problems is included in lesson. • v. Note: Converting between degrees and radians is included in lesson.
HSG-GPE.A.1	• i. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. • ii. Note: Graphing circles from their equation is included in lesson.
HSG-GPE.A.2	• Derive the equation of a parabola given a focus and directrix.
HSG-GPE.A.3	• Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Unit 7: Measurement	
HSG-GPE.B.7	• Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
6.G-A.1	• i. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. • ii. Note: Determining the area and arc length of sectors is included in lesson.
7.G-B.6	• Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
HSG-GMD.A.1	• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
HSG-GMD.A.2	• Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

HSG-GMD.A.3	• i. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. • ii. Note: Prisms are included in lesson.
HSG-GMD.B.4	• i. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
7.G-A.1	• Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
HSG-MG.A.1	• Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
HSG-MG.A.2	• Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
HSG-MG.A.3	• Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
Unit 8: Reasoning and Proofs	
HSG-GPE.B.5	• Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
HSG-CO.C.9	• Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
HSG-CO.C.10	• i. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; • ii. base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
HSG-SRT.B.4	• i. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, • ii. and conversely; the Pythagorean Theorem proved using triangle similarity.
HSG-SRT.B.5	• i. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. • ii. Note: Criteria include SAS, ASA, SSS, AAS, and HL.
HSG-CO.C.11	• i. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. • ii. Note: Using proofs to determine if a quadrilateral is a parallelogram, rectangle, square, or rhombus is included in lesson.

HSG-C.A.1	• Prove that all circles are similar.
HSG-GPE.B.4	• Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,√3) lies on the circle centered at the origin and containing the point (0,2).
HSG-C.A.2	• Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
HSG-C.A.3	• Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Unit 9: Statistics	
S-ID.A.1	• Represent data with plots on the real number line (dot plots, histograms, and box plots).
S-ID.A.2	• Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S-ID.A.3	• Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S-ID.A.4	• Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
S-ID.B.5	• i. Summarize categorical data for two categories in two-way frequency tables. • ii. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S-ID.B.6	• Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
S-ID.C.7	• Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S-ID.C.8	• Compute (using technology) and interpret the correlation coefficient of a linear fit.
S-ID.C.9	• Distinguish between correlation and causation.

S-IC.A.1	• Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S-IC.A.2	• Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?
S-IC.B.3	• Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S-IC.B.4	• Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S-IC.B.5	• Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S-IC.B.6	• Evaluate reports based on data.
Unit 10: Probability	
HSS-CP.A.1	• Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
HSS-CP.A.2	• i. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. • ii. Note: lesson also includes computing probabilities with or without replacement.
HSS-CP.A.3	• Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
HSS-CP.A.4	• Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
HSS-CP.A.5	• Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
HSS-CP.B.6	• Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model.

HSS-CP.B.7	• Apply the Addition Rule, *P(A* or *B) = P(A) + P(B) – P(A* and *B)*, and interpret the answer in terms of the model.
HSS-CP.B.8	• (+) Apply the general Multiplication Rule in a uniform probability model, *P(A* and *B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.
HSS-CP.B.9	• (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
HSS-MD.B.6	• (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
HSS-MD.B.7	• (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
HSS-MD.A.1	• (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
HSS-MD.A.2	• (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
HSS-MD.A.3	• (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
HSS-MD.A.4	• (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
HSS-MD.B.5	• (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fastfood restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*